

Integrable Discretisations of the Nonlinear Schrödinger Equation on Grassmann Algebras

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Abstract:

Integrable discretisations for a class of coupled nonlinear Schrödinger (NLS) type of equations are presented. The class corresponds to a Lax operator with entries in a Grassmann algebra.

Elementary Darboux transformations are constructed. As a result, Grassmann generalisations of the Toda lattice and the NLS dressing chain are obtained.

Furthermore, the compatibility (Bianchi commutativity) of these Darboux transformations leads to integrable Grassmann generalisations of the difference Toda and NLS equations.

The resulting discrete systems will have Lax pairs provided by the set of two consistent Darboux transformations. The corresponding Bäcklund transformations represent symmetries of the discrete (difference systems) and formal diagonalisations of the Darboux transformations provide generating functions of integrals of motion.